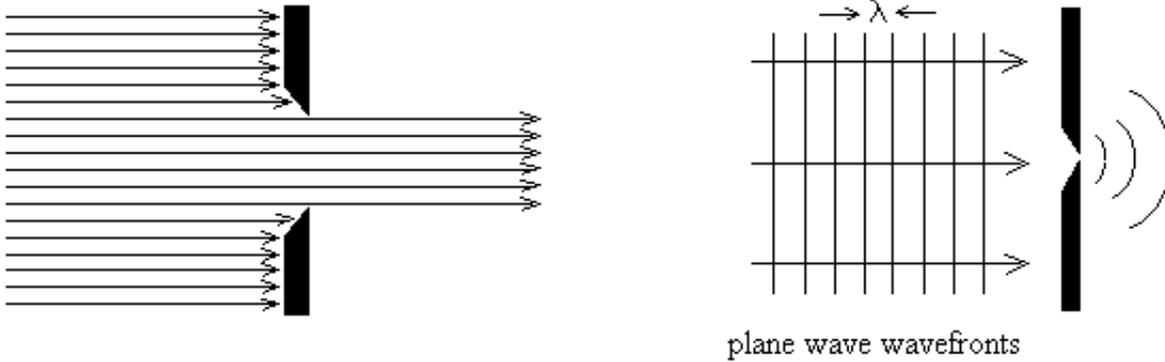
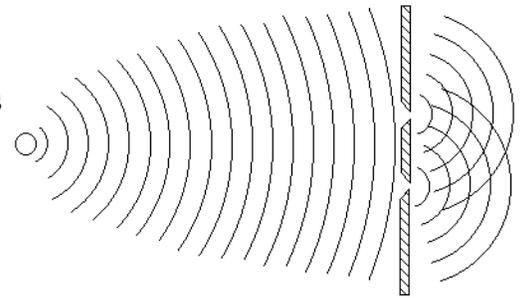


Lab 5: Diffraction and Interference

Light is a wave, an electromagnetic wave, and under the proper circumstances, it exhibits wave phenomena, such as constructive and destructive interference. The wavelength of light is about $600\text{nm} = 0.0006\text{mm}$, and this wavelength λ sets the scale for the appearance of wave-like effects. For instance, if a broad beam of light partly passes through a wide slit, a slit which is very large compared to λ , then the wave effects are negligible, the light acts like a ray, and the slit casts a geometrical shadow. However, if the slit is small enough, then the wave properties of light become apparent and a diffraction pattern is seen projected onto a screen illuminated by the light from the narrow slit.



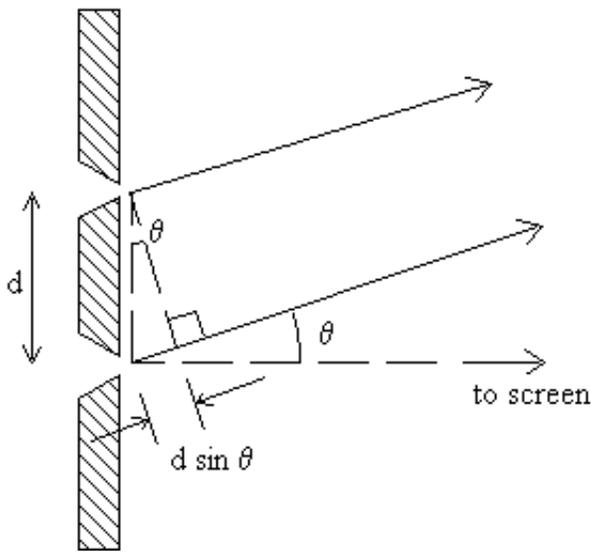
We now ponder the light from two *coherent* light sources a distance d apart. Coherent sources emit light waves that are *in phase*, or in sync. If we think of light like a water wave, we can imagine that the two sources each emit an identical succession of wave crests and troughs, with the both emitting crests at the same. One way to create such coherent sources is to illuminate a pair of narrow slits with a distant light source.



Consider the light rays from the two coherent point sources made from *infinitesimal* slits a distance d apart (diagram below). We assume that the sources are emitting monochromatic light of wavelength λ . The rays are emitted in all forward directions, but let us concentrate on only the rays that are emitted in a direction θ toward a distant screen (θ measured from the normal to the screen, diagram below). One of these rays has further to travel to reach the screen, and the *path difference* is given by $d \sin \theta$. If this path difference is exactly one wavelength λ or an integer number of wavelengths, then the two waves arrive at the screen in phase and there is constructive interference, resulting in a bright area on the screen. If the path difference is $\frac{1}{2}\lambda$, or $\frac{3}{2}\lambda$, etc., then there is destructive interference, resulting in a dark area on the screen.

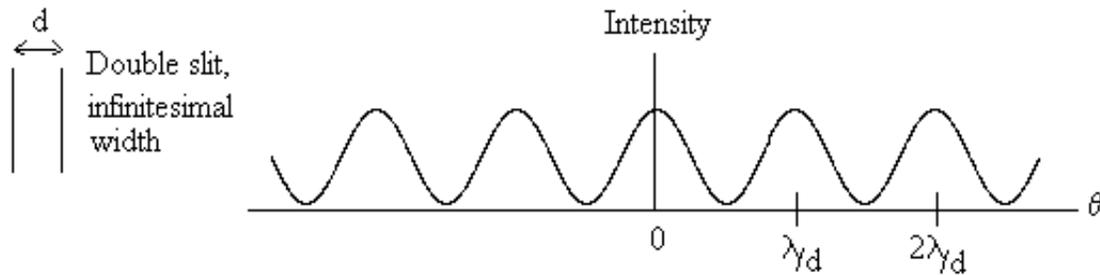
$$\left. \begin{array}{l} \text{Bright: } d \sin \theta = m\lambda \\ \text{Dark: } d \sin \theta = (m + \frac{1}{2})\lambda \end{array} \right\} m = 0, \pm 1, \pm 2, \dots$$

If θ is small, then $\sin \theta \cong \theta$ (in radians), and maxima occur on the screen at $\theta = m \frac{\lambda}{d}$; minima occur at

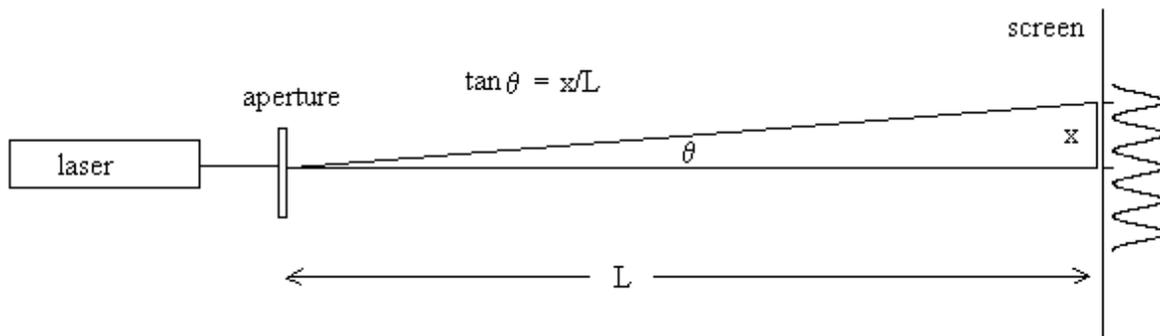


$$\theta = \left(m + \frac{1}{2}\right) \frac{\lambda}{d}$$

A complete analysis (not show here) yields a pattern of intensity vs. angle that looks like:



As shown below, the angle θ (measured from the center of the screen) is related to the distance x measured on the screen by $\tan \theta = \frac{x}{L}$, where L is the distance from the screen to the source of light (the aperture).

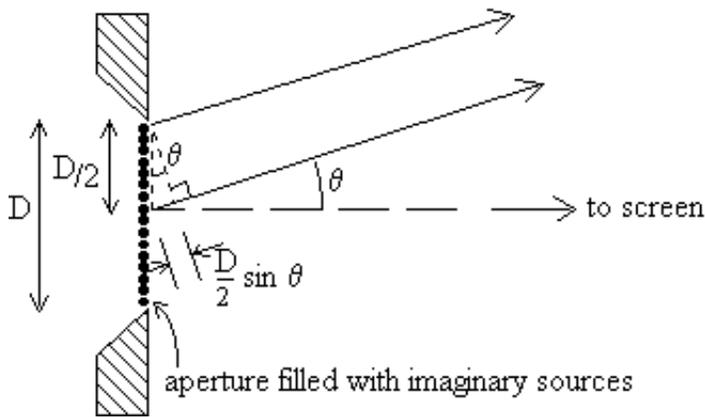


If the angle θ is small (less than a few degrees), then to an excellent approximation,

$$\sin \theta \cong \tan \theta \cong \theta \text{ (in radians)} \text{ and so the locations of the interference maxima are given by } \frac{x}{L} = m \frac{\lambda}{d}$$

The uniform 2-slit interference pattern shown above is seldom observed in practice, because real slits always have finite width (not an infinitesimal width). We now ask: what is the intensity pattern from a single slit of finite width D ? *Huygens' Principle* states that the light coming from an aperture is the same as the light that would come from a collection of coherent point sources filling the space of the aperture. To see what pattern the entire array produces, consider first just two of these imaginary sources: one at the edge of the slit and one in the center. These two sources are separated by a distance $D/2$.

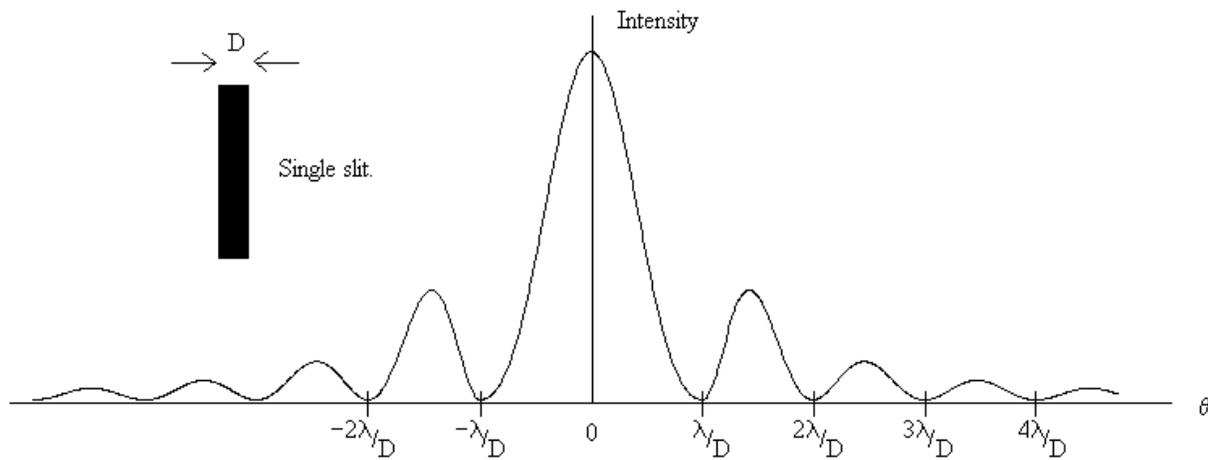
The path difference for the rays from these two sources, going to the screen at an angle θ , is $\frac{D}{2} \sin \theta$, and



these rays will interfere destructively if $\frac{D}{2} \sin \theta = \frac{\lambda}{2}$.
 . But the same can be said for every pair of sources separated by $D/2$. Consequently, the rays from all the sources filling the aperture cancel in pairs, producing zero intensity on the screen when $\frac{D}{2} \sin \theta = \frac{\lambda}{2}$ or, if θ is small,

$$\theta = \frac{\lambda}{D} . \text{ (First minimum in single slit pattern.)}$$

A complete analysis (too complicated to show here) yields an intensity pattern, called a *diffraction* pattern, on the screen that looks like...

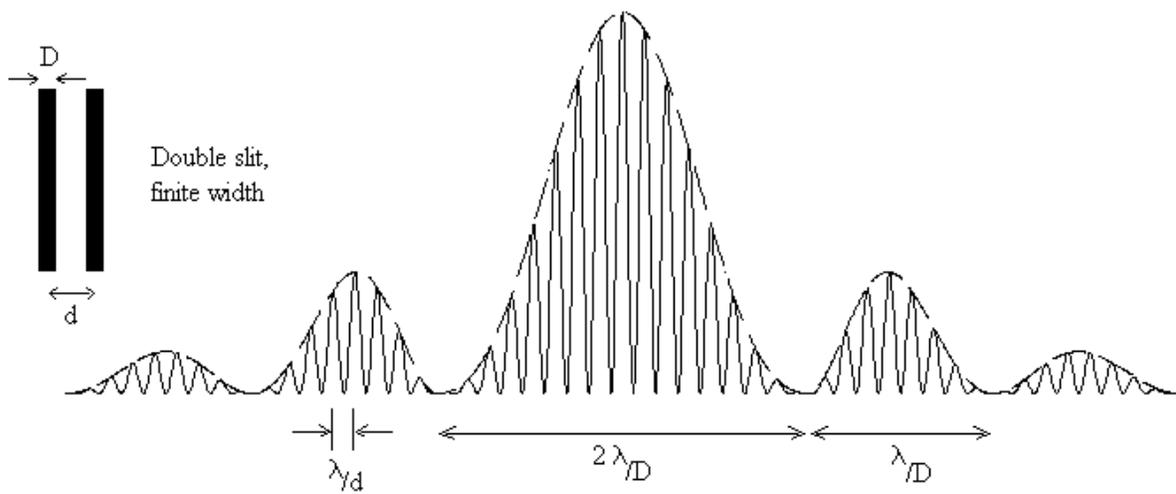


[The central maximum is actually much higher than shown here. It was reduced by a factor of 6, for clarity.]
 The single slit diffraction pattern has minima at

$$\theta = \pm \frac{\lambda}{D}, \pm \frac{2\lambda}{D}, \pm \frac{3\lambda}{D}, \dots \text{ (Minima of single slit pattern.)}$$

So the separation of minima is λ/D , except for the first minima on either side of the central maximum, which are separated by $2\lambda/D$. If x is the distance on the screen between minima, then $\theta = \frac{x}{L} = \frac{\lambda}{D}$.

When the aperture consists of two *finite* slits, each of width D , separated by a distance d , then the intensity pattern is a combination of both the single-slit pattern and the double slit pattern: the amplitude of the two slit interference pattern is modulated by a single slit diffraction pattern:



In this full pattern, the finely spaced interference maxima are spaced $\Delta\theta = \frac{\lambda}{d}$ apart, while the more widely spaced minima of the single-slit diffraction pattern are separated by $\frac{\lambda}{D}$ or $\frac{2\lambda}{D}$. Note that an interference maximum can be wiped out if it coincides with a diffraction minimum.

An aperture consisting of many slits of uniform width and spacing is called a *diffraction grating*. Using exactly the same arguments as above, one can show that a diffraction grating, whose slit separation is d , produces maxima with a separation of $\Delta\theta = \frac{\lambda}{d}$, the same as a double slit, except the maxima from a grating are very cleanly separated and sharply peaked. The more slits in the grating, the more sharply peaked the maxima.

The resolution of an optical instrument such as a camera, a telescope or the human eye is defined as the smallest angle which the instrument can resolve. Diffraction effects limit the resolution of any optical

instrument to $\theta \cong \frac{\lambda}{D}$, where λ is the wavelength of the light used, and D is the diameter of the main light-gathering optical element (the objective lens of a telescope, the pupil of the human eye, etc.). Many texts use

the formula $\theta = 1.22 \frac{\lambda}{D}$, called the Rayleigh criterion, but the minimum angle of resolution θ is difficult to measure precisely, and in practice the factor 1.22 is not experimentally significant.

Experiment

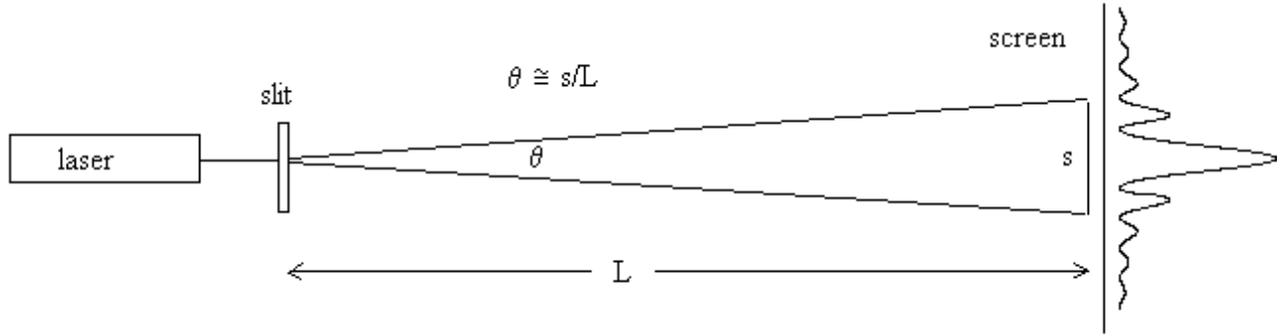
The light source in parts 1 and 2 of this experiment is a He-Ne laser which produces a monochromatic beam with a wavelength of $\lambda = 632.8 \text{ nm}$ and a beam diameter of about 1 mm . The power output of our lasers is about 1 mW , a small amount, but still enough to damage your retina if you look directly into the beam.

The aperture consists of an opaque photographic negative, containing several single, double, and multiple slits. The arrangement of slits on the plate is shown here. The numbers are those given by the manufacturer and are *not* always accurate. (This particular plate is often called a Cornell Plate, since apertures of this kind were first used at Cornell University.)

1 1.43 -	15 0.05 0.176	1 0.05 -	1 0.107 -
1 0.755	30 0.05 0.088	2 0.05 0.132	2 0.10 0.175
1 0.345	75 0.01 0.034	3 0.05 0.132	2 0.10 0.35
1 0.195	37 0.027 0.066	4 0.05 0.132	2 0.10 0.70
1 0.108	20 0.044 0.088	10 0.05 0.132	2 0.10 1.40

Number of lines N.
Width D in mm.
Spacing d in mm.

Place the Cornell plate in its holder and mount it on the optical bench a few centimeters in front of the laser. Place a piece of white paper in the clipboard and place it at the far end of the bench.. Then shine the laser through the various apertures and observe the diffraction patterns. Spend a few minutes exploring. Note the appearance of the single, double, and multiple slit patterns. In particular, observe how the single slit pattern changes as you vary the width of the slit.

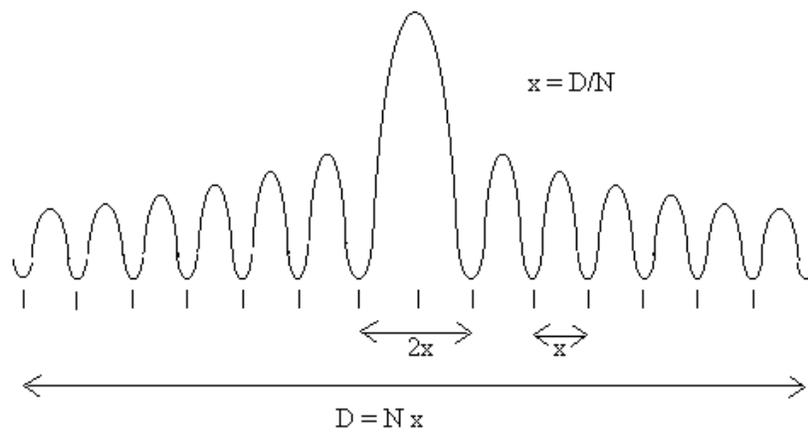


This diagram shows the relation between the width s of some feature (any feature) on the screen, the angular width θ of that feature, and the distance L from the aperture to the screen.

Part 1. Single Slits

$$\theta = \frac{x}{L} = \frac{\lambda}{D}$$

In this part we will test the relation $\theta = \frac{x}{L} = \frac{\lambda}{D}$, where x is the separation of minima on the screen in the single slit diffraction pattern. Measure the distance L from the slit to the screen. Observe the diffraction pattern on your paper screen for each of the three smallest single slits(the three with approximate widths D = 0.10, 0.20, 0.35mm). With a pencil, mark the positions of as many of the minima that you can see and measure the spacing x between adjacent minima on the screen, for each of the three slits. To do this most accurately, measure the width of the entire pattern and divide by the number of maxima in the pattern (central max counts as two!). From your measurements, using the wavelength $\lambda = 632.8\text{nm}$, compute the slit width D for each of the three slits and compare with the manufacturer's widths.



Part 2. Double Slit

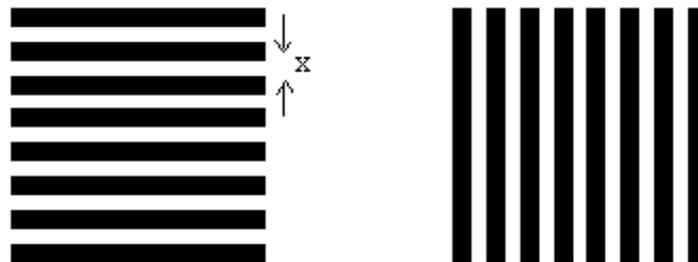
Repeat the above procedure with the one double slit aperture which has a nominal slit separation of 0.35 mm (the one in the middle of the right most column on the Cornell plate). Using the wavelength $\lambda = 632.8\text{nm}$, and make measurements of the diffraction pattern which allow you to compute d , the separation of the slits. Compare your value for d with the manufacturer's.

Part 3. Resolving power of the human eye

In this part, you will measure the resolving power of your eyes to see how close your vision is to "perfect", that is, diffraction-limited performance.

First, measure the diameter D of the pupil of your eye (under normal room-light conditions). With one eye open, look closely at the image of your pupil in a mirror and measure your pupil's diameter with a clear plastic ruler placed on the mirror or over your eye.

Next, measure the angular resolution of your eye with the following procedure: Your partner will stand on a "zero position" mark on the floor some distance away and will hold up an eye-test chart for you to view with both eyes open. The test chart consists of an array of vertical bars and an array of horizontal bars, as shown below. Beyond a certain distance, the human eye cannot resolve the bars and the arrays appear to be gray blotches rather than black stripes on a white background.

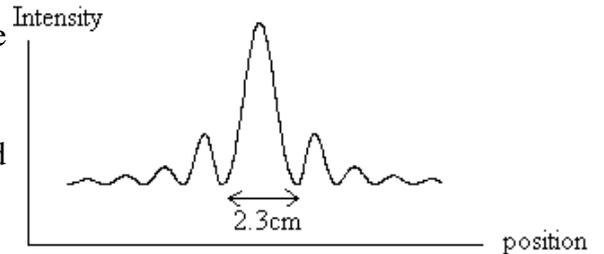


Begin by standing so far from your partner that the chart cannot possibly be resolved, and then signal your partner to hold up the test chart. The chart is held up with either the vertical bars or the horizontal bars to your right, but you will not know which orientation is used. Slowly, approach your partner and when you believe you can resolve the bars, indicate with hand signals which side of the chart has the horizontal bars. Repeat this test several times, varying your distance somewhat. Each time you "read" the chart, your partner

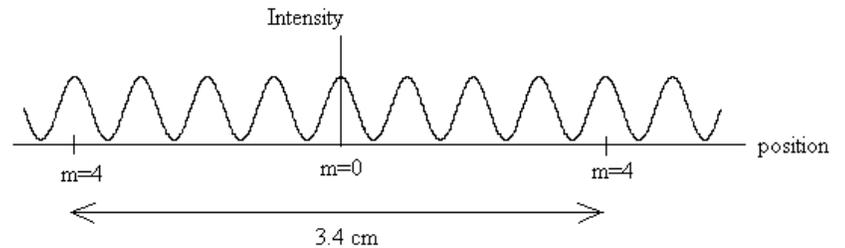
should give the thumbs-up(correct) or thumbs-down(incorrect) signal and then you should turn away briefly while your partner randomly rearranges the chart for the next test. Find the maximum distance L (marked on the floor) at which you can consistently read the chart correctly. The separation x of the bars is marked on the chart. The angle α which you can resolve is then $\alpha = x/L$. Report this angle in radians, degrees, and in arcminutes (60 arcminutes = 1 degree). Compare this angle with the theoretical diffraction-limited resolution of $\alpha = \lambda/D$. Use $\lambda = 550\text{nm}$ (the middle of the visible spectrum) in your calculations. Also report the results of your partners' eye tests. How close is your vision and your partners' to "perfect"?

Prelab questions (due at the beginning of the lab period)

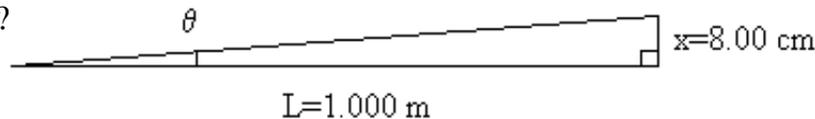
1. A He-Ne laser (wavelength $\lambda = 633\text{nm}$) shines through a single slit of unknown width D onto a screen 2.00 m away from the slit. The distance on the screen between the $m=1$ minima on either side of the central maximum of the diffraction pattern is measured and is found to be 2.3cm . What is the width D of the slit?



2. A He-Ne laser (wavelength $\lambda = 633\text{nm}$) shines through a double slit of unknown separation d onto a screen 2.00 m away from the slit. The distance on the screen between the $m=4$ maxima on either side of the central maximum of the two-slit diffraction pattern is measured and is found to be 3.4 cm . What is the separation d of the two slits?



3. For the triangle shown below, what is the angle α in radians and in degrees? What would be your answer if you made the small angle approximation ($\sin \theta \cong \tan \theta \cong \theta$)? What would be your % error in α if you made the small angle approximation?



4. What is Huygen's Principle?

5. True or False: it is perfectly safe to stare into a laser.